Review Guide I
Math 161 - Fall 2014

1 Functions and Their Representations

Topics covered

- Definition of a function, Domain, Codomain, range
- Representation of a function (graphically, algebraically)
- Symmetry of a function
- Section 1.1: Example 3, 7

2 Catalog of Essential Functions

Topics covered

- Linear functions, Piecewise functions.
- Graphs and Domains of the functions, x^n, $\sqrt[n]{x}$ where n is positive integer, $\frac{1}{x}$, $\sin(x)$, $\cos(x)$, $|x|$
- Graphing function using transformation of functions
- Combining functions $f+g$, $f-g$, $f \cdot g$, $\frac{f}{g}$
- Function Composition
- Section 1.2: Example 2, 5, 6

3 Limits

Topics covered Topics covered

- Determining limits from a graph, including one-sided limits
- Limit Laws 1-10, Direct Substitution Property
- Squeeze Theorem (Section 1.4, pg 41)
- Section 1.3: Example 7
- Section 1.4: Example 4, 5, 7, 9, 10

4 Infinite Limits

Topics covered

- Infinite Limits, Vertical Assymptotes
- Limits at Infinity, Horizontal Assymptotes
- Infinity limits at Infinity
- Section 1.6: Example 1, 5, 6

5 Continuity

Topics covered

- Definition of continuity, types of discontinuities, Continuity from left/right
- Intermediate Value Theorem
- Section 1.5: Example 2, 6, 7, 8

6 Practice Problems

1. Fill in the blanks

   (a) A function f is an even function if for every x in the domain of f, $f(x) =$_____. The graph of the even function is symmetric with respect to the _____. If (a, b) is on the graph of $f(x)$, then so is _____.

   (b) $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} =$_____ and $\lim_{\theta \to 0} \frac{\tan \theta}{\theta} =$_____.

   (c) For a positive integer n, $\lim_{x \to \infty} \frac{1}{x^n} =$_____ and $\lim_{x \to -\infty} \frac{1}{x^n} =$_____.

   (d) For a positive integer n, $\lim_{x \to \infty} x^n =$_____.

   (e) For a positive integer $\lim_{x \to -\infty} x^n = \begin{cases} \text{\_\_\_\_\_} & \text{if } n \text{ is odd} \\ \text{\_\_\_\_\_} & \text{if } n \text{ is even} \end{cases}$

   (f) A function f is continuous at a point a if $\lim_{x \to a} f(x) =$_____ .

   (g) The following types of functions are continuous at every number in their domain: _____, _____, _____, _____. (See Page 51)

2. Determine if the following functions are odd, even or neither

   (a) $f(x) = x^4 + x^2 + 1$

   (b) $f(x) = x^3 - x$

   (c) $f(x) = \dfrac{1}{\sqrt{1 + x^2}}$

3. Graph the following functions by starting with the graph of one of the standard functions and then applying the appropriate transformations.

   (a) $f(x) = -(x - 3)^2 + 1$

   (b) $g(x) = |x + 2| - 3$

4. Let $f(x) = \sqrt{x - 7}$ and $g(x) = x^3 + 1$

   (a) Determine the domain of $f(x), g(x)$.

   (b) Find $f \circ f(x)$ and $f \circ g(x)$

5. Given $F(x) = \sqrt[8]{x^4 + 3}$, find functions f, g, h such that $F = f \circ g \circ h$

6. Determine the following limits

   (a) $\lim_{x\to 2} \dfrac{x^2+6x+3}{x^2-9}$

   (b) $\lim_{x\to 0} \dfrac{\sin(8x)}{9x}$

   (c) $\lim_{x\to 4} \dfrac{x^2-8x+16}{x^2-6x+8}$

   (d) $\lim_{x\to -3} \dfrac{2x}{x^2-9}$

   (e) $\lim_{x\to 0} \dfrac{\sqrt{x^2+25}-5}{x}$

   (f) $\lim_{x\to \infty} \dfrac{x^3-4x^2+6}{x^2-6x^3+8x+9}$

7. (a) State the Squeeze Theorem.

   (b) Use the Squeeze Theorem to show that $\lim_{x\to 0} x^4 \sin\left(\dfrac{1}{x}\right) = 0$

8. Determine the horizontal and vertical assymptotes of the following functions:

   (a) $f(x) = x^2 + 3x + 2$

   (b) $f(x) = \dfrac{x^2-4x-5}{x^2-6x+5}$

9. On what interval are the following functions continuous. Justify your answers:

   (a) $f(x) = x^{69} + x^{32} + x^{11} - x$

   (b) $g(x) = \dfrac{x^2-4x+3}{x-1}$

   (c) $h(x) = \dfrac{1}{\sqrt{1+x^2}}$

   (d) $k(x) = \sin(x^3+2x+1)$

   (e) $p(x) = \sqrt{x-2} + \sqrt[4]{5-x}$

10. (a) State the Intermediate Value Theorem

    (b) Use the Intermediate Value Theorem to show that the equation $x^4 + x^3 + x^2 + x - 2 = 0$ has a root between -1 and 1.